EXPLANATION OF THE RHIC HBT PUZZLE BY A GRANULAR SOURCE OF QUARK-GLUON PLASMA DROPLETS

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We present a review on the explanation of the RHIC HBT puzzle by a granular pion-emitting source of quark-gluon plasma droplets. The evolution of the droplet is described by relativistic hydrodynamics with an equation of state suggested by lattice gauge results. The granular source evolution is obtained by superposing all of the evolutions of individual droplets. Pions are assumed to be emitted thermally from the droplets at the freeze-out configuration characterized by a freeze-out temperature T_f. We find that the average particle emission time scales with the initial radius of the droplet. Pions will be emitted earlier if the droplet radius is smaller. An earlier emission time will lead to a smaller extracted HBT radius $R_{\rm out}$, while the extracted HBT radius $R_{\rm side}$ is determined by the scale of the distribution of the droplet centers. However, a collective expansion of the droplets can further decrease $R_{\rm out}$. As a result, the value of $R_{\rm out}/R_{\rm side}$ can be close to, or even less than 1 for the granular source of QGP droplets.

1. Introduction

HBT (Hanbury-Brown-Twiss) interferometry is an indispensable tool to study the space-time structure of the particle-emitting source produced in high energy heavy ion collisions[1,2,3]. The experimental pion HBT measurements at RHIC give the ratio of $R_{\rm out}/R_{\rm side} \approx 1$ [4,5,6,7], which is much smaller than many earlier theoretical expectations. Such a discrepancy between theory and experiment is referred to as the RHIC HBT puzzle[2,3,8,9,10,11]. On the other hand, hydrodynamical calculations give reasonably good descriptions of the elliptic flow, which has been considered as an evidence for a strongly-coupled quark-gluon plasma[12,13,14,15,16]. The resolution of the HBT puzzle is important in finding out why traditional hydrodynamics succeed in explaining the elliptic flow but fails in explaining the HBT radii.

Traditional studies of the hydrodynamics of the evolving fluid assume a single contiguous blob of matter under expansion, with a relatively smooth initial and final density distributions. Initial transverse density fluctuations and hydrodynamical instabilities have been neglected but their inclusion may lead to "multi-fragmentation" in the form of large scale final-state density fluctuations and the formation of granular droplets. It is useful to explore the consequences of the occurrence of granular droplets.

Previously we propose a granular model to explain the HBT puzzle[17,18]. We would like to review here the important ingredients which enters into the resolution of the puzzle. Further suggestions of using single-event HBT interferometry to search for signatures of the granular source can be found in Refs. [19,20,21].

2. Evolution of a QGP droplet

Based on the recent results of high-energy heavy-ion collisions at RHIC, the early matter produced in the collisions may be a strongly-coupled QGP (sQGP), which has a very high energy density and reaches local thermalization within about 1 fm/c[12,13,14,15,16]. The expansion of the matter after that time may be unstable. Many effects, such as the large fluctuations of the initial transverse energy density[22,23,24], the sausage instability[25], and possible phase transition[26], may lead to the fragmentation of the system and the formation of many spherical droplets due to the surface tension of the QGP [18,19,21].

To describe the evolution of a droplet, we use relativistic hydrodynamics where the energy momentum tensor of a thermalized fluid element in the center-of-mass frame of the droplet is[27]

$$T^{\mu\nu}(x') = \big[\epsilon(x') + p(x')\big] u^{\mu}(x') u^{\nu}(x') - p(x') g^{\mu\nu}\,, \tag{1}$$

x' is the space-time coordinate of the fluid element in the center-of-mass frame, ϵ, p, and $u^{\mu} = \gamma(1, \boldsymbol{v})$ are the energy density, pressure, and 4-velocity of the element, and $g^{\mu\nu}$ is the metric tensor. With the local conservation of energy and momentum, one can obtain the equations for spherical geometry as[8]

$$\partial_t E + \partial_r[(E+p)v] = -F\,, \tag{2}$$

$$\partial_t M + \partial_r(Mv + p) = -G\,, \tag{3}$$

where $E \equiv T^{00}$, $M \equiv T^{0r}$, $F = 2v(E+p)/r$, $G = 2vM/r$.

In the equations of motion (2) and (3) there are three unknown functions ϵ, p, v. In order to obtain the solution of the equations of motion, we need an equation of state which gives a relation $p(\epsilon)$ between p and ϵ [8,28]. At RHIC energy, the system undergoes a transition from the QGP phase to hadronic phase. As the net baryon density in the central rapidity region is much smaller than the energy density of the produced matter (here presumed to be QGP), the baryon density of the system in the center rapidity region can be neglected. Lattice gauge results suggest the entropy density of the system as a function of temperature as[8,28,29,30]

$$\frac{s(T)}{s_c} = \left(\frac{T}{T_c}\right)^3 \left[1 + \frac{d_Q - d_H}{d_Q + d_H} \tanh\left(\frac{T - T_c}{\Delta T}\right)\right], \tag{4}$$

where s_c is the entropy density at the transition temperature T_c, d_Q and d_H are the degrees of freedom in the QGP phase and the hadronic phase, and ΔT is the width of the transition. The thermodynamical relations among p, ϵ, and s in this case are

$$-sdT + dp = 0\,, \qquad \epsilon = Ts - p\,. \tag{5}$$

From these thermodynamical relations and Eq. (4), we can obtain the equation of state $p(\epsilon)$.

Using the HLLE scheme[31,32] and Sod's operator splitting method[33], one can obtain the solution of Eqs. (2) and (3)[8,28,17], after knowing the equation of state and initial conditions. We assume that the droplet has a uniform initial energy density ϵ_0 within a sphere with radius r_d, and has a zero initial velocity in its center-of-mass frame. Figs. 1(a) and (b) show the temperature profiles and isotherms for the droplet. In our calculations, we take the parameters of the equation of state as $d_Q = 37$, $d_H = 3$, $T_c = 165$ MeV, and $\Delta T = 0.05\, T_c$, and take the initial energy density $\epsilon_0 = 3.75 T_c s_c$, which is about two times of the density of quark matter at T_c [8,28].

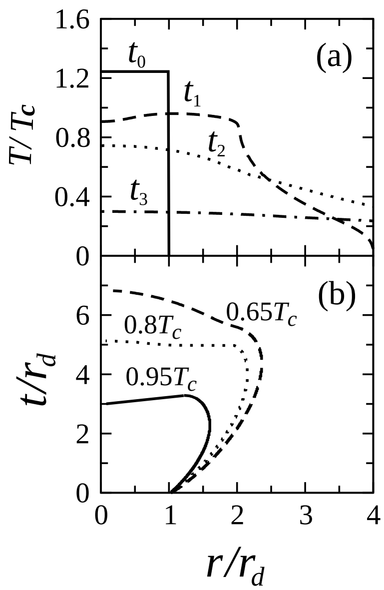

Fig. 1. (a) Temperature profile and (b) isotherms for the droplet. Here, $t_n = 3n\lambda r_d$ and $\lambda = 0.99$.

3. Granular Source of QGP droplets

If we assume that the final pions are emitted from the droplet at the freeze-out configuration characterized by a freeze-out temperature T_f, we can see from figure 1(b) that the the average particle emission time scales with the initial radius of the droplet r_d. In HBT interferometry, the radius $R_{\rm side}$ is related to the spatial size of the particle-emitting source and the radius $R_{\rm out}$ is related not only to the source spatial size but also to the lifetime of the source[2,3,34,35]. A long lifetime of the source will lead to a large $R_{\rm out}$[2,3,34,35]. From the hydrodynamical solution in figure 1(b), both the average freeze-out time and freeze-out radial distance increase with r_d for a single droplet source. As a consequence, $R_{\rm out}/R_{\rm side}$ is insensitive[17] to the values r_d. The value of $R_{\rm out}/R_{\rm side}$ for the single droplet source[17] is about 3 [17], much larger than the observed values[4,5,6,7].

The RHIC HBT puzzle of $R_{\rm out}/R_{\rm side} \sim 1$ suggests that the pion emitting time may be very short. In order to explain the HBT puzzle, we consider a granular source of N_d QGP droplets distributed in a distribution $D(X_d)$ as illustrated in Fig. 2(a). We assume that the evolution of the granular source is simply the superposition of the evolutions of individual droplets with the same initial conditions. As the average freeze-out time is proportional to the initial radius of the droplet, the freeze-out time and $R_{\rm out}$ decreases if the initial radius of the droplet decreases. On the other hand, $R_{\rm side}$ increases if the width of the droplet spatial distribution $D(X_d)$ increases. A variation of the droplet size and the width of droplet spatial distribution can result in $R_{\rm out}$ nearly equal to $R_{\rm side}$ [[17]]. Furthermore, if the granular source has a collective expansion the HBT will measure the size of the region of the ellipse in figure 2(b)[2,3]. In this case the value of $R_{\rm out}/R_{\rm side}$ decreases even further and may be smaller than unity[17].

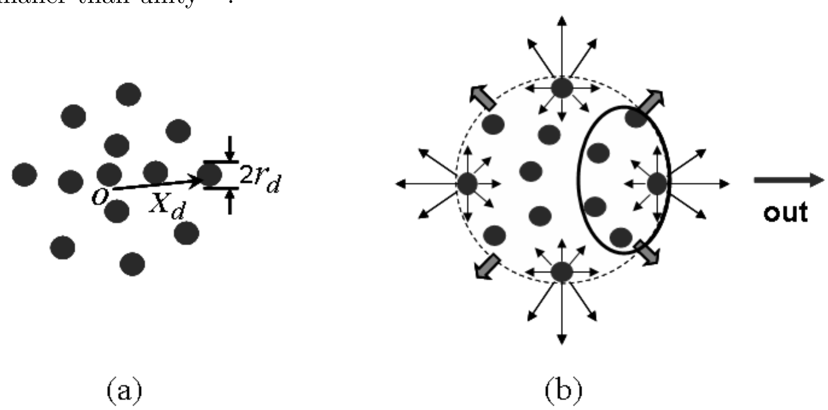

Fig. 2. (a) Static Granular source of the QGP droplets. (b) Expanding granular source of the QGP droplets.

We assume that the droplets are initially distributed in a short cylinder of length $2\mathcal{R}_z$ along the beam direction (z direction) with an initial transverse spatial distribution up to a radius $\mathcal{R}_t$,

$$\frac{dP_d}{2\pi\rho\, d\rho\, dz} \propto [1-\exp(-\rho^2/\Delta\mathcal{R}_t^2)]\theta(\mathcal{R}_t-\rho)\,\theta(\mathcal{R}_z-|z|)\,, \tag{6}$$

where $\rho = \sqrt{x^2+y^2}$ and z are the coordinates of the center of a droplet, and $\Delta\mathcal{R}_t$ describes a shell-type radial distribution. Because of the early thermalization and the anisotropic pressure gradient, the droplets will acquire anisotropic initial velocities. We therefore consider a granular source of the the QGP droplets with an anisotropic velocity distribution and the velocity of a droplet depends on the initial coordinates of the droplet center, $(r_1, r_2, r_3) = (x, y, z)$, in the form

$$\boldsymbol{\beta}_i = a_i\, \mathrm{sign}(r_i)\left(\frac{|r_i|}{\mathcal{R}_i}\right)^{b_i}, \qquad i = 1, 2, \text{ and } 3, \tag{7}$$

where a_i describes the magnitude of the anisotropic expansion, sign(r_i) denotes the sign of r_i, and $b_i\,(b_{\scriptscriptstyle T}, b_z)$ are the exponential power parameters that describe the variation of the velocities with r_i. In our calculations the velocity parameters are taken as $a_x = 0.415$, $a_y = 0.315$, $a_z = 0.850$, $b_x = b_y = 0.42$, and $b_z = 0.03$, by comparing with the experimental data of pion transverse momentum spectral[36] and elliptic flow[37] in $\sqrt{s_{NN}} = 200$ GeV Au + Au collisions at RHIC. We take the freeze-out temperature as $T_f = 0.95T_c$ in our calculations.

4. HBT Results of the Granular Source of QGP Droplets

The two-particle Bose-Einstein correlation function is defined as the ratio of the two-particle momentum distribution $P(p_1, p_2)$ relative to the the product of the single-particle momentum distribution $P(p_1)P(p_2)$. For a chaotic pion-emitting source, $P(p_i)$ $(i = 1, 2)$, and $P(p_1, p_2)$ can be expressed as[1]

$$P(p_i) = \sum_{X_i} A^2(p_i, X_i)\,, \qquad P(p_1, p_2) = \sum_{X_1, X_2} \Big|\Phi(p_1, p_2; X_1, X_2)\Big|^2, \tag{8}$$

where $A(p_i, X_i)$ is the magnitude of the amplitude for emitting a pion with 4-momentum $p_i = (\boldsymbol{p}_i, E_i)$ in the laboratory frame at X_i and is given by the Bose-Einstein distribution with freeze-out temperature T_f in the local rest frame of the source point. $\Phi(p_1, p_2; X_1, X_2)$ is the two-pion wave function. Neglecting the absorption of the emitted pions by other droplets, $\Phi(p_1, p_2; X_1, X_2)$ is simply

$$\Phi(p_1, p_2; X_1, X_2) = \frac{1}{\sqrt{2}}\Big[A(p_1, X_1)A(p_2, X_2)e^{ip_1 \cdot X_1 + ip_2 \cdot X_2} + (X_1 \leftrightarrow X_2)\Big]. \tag{9}$$

Using the components of "out", "side", and "long"[34,35] of the relative momentum of the two pions, $q = |\mathbf{p_1} - \mathbf{p_2}|$, as variables, we can construct the correlation function $C(q_{\rm out}, q_{\rm side}, q_{\rm long})$ from $P(p_1, p_2)$ and $P(p_1)P(p_2)$ by picking pion pairs from the granular source and summing over $\mathbf{p_1}$ and $\mathbf{p_2}$ for each $(q_{\rm out}, q_{\rm side}, q_{\rm long})$ bin[17]. The HBT radii $R_{\rm out}$, $R_{\rm side}$, and $R_{\rm long}$ can then be extracted by fitting the calculated correlation function $C(q_{\rm out}, q_{\rm side}, q_{\rm long})$ with the following parametrized correlation function

$$C(q_{\rm out}, q_{\rm side}, q_{\rm long}) = 1 + \lambda\, e^{-q_{\rm out}^2 R_{\rm out}^2 - q_{\rm side}^2 R_{\rm side}^2 - q_{\rm long}^2 R_{\rm long}^2}\,. \tag{10}$$

Figure 3 shows the theoretical two-pion correlation functions for the granular source. The top figures give the results for the average pion transverse momentum of a pion pair, K_T, less than 400 MeV/c, and the bottom figures for $K_T > 400$ MeV/c. In our calculations the size parameters of the granular sources are taken to be $\mathcal{R}_t = 8.8$ fm, $\Delta\mathcal{R}_t = 3.5$ fm, $\mathcal{R}_z = 7.0$ fm, and $r_d = 1.3$ fm. The number of droplet N_d is taken to be 40 in our calculations. The left panels in Fig. 4 give the extracted two-pion HBT radii for the granular source as a function of K_T. The symbols of circle and down-triangle are for $\Delta\mathcal{R}_t = 0.35$ fm and $\Delta\mathcal{R}_t = 0$ fm, respectively. The experimental PHENIX results[6], and STAR results[7] are shown on the right panels. The curve gives the theoretical results for $\Delta\mathcal{R}_t = 0.35$ fm. For our theoretical HBT calculations, we use a cut for particle pseudo-rapidity region $|\eta| < 0.35$, the same as in the PHENIX experiments[6]. We find that if we increase the parameter r_d, the HBT radii $R_{\rm out}$ and $R_{\rm long}$ will increase. And if we

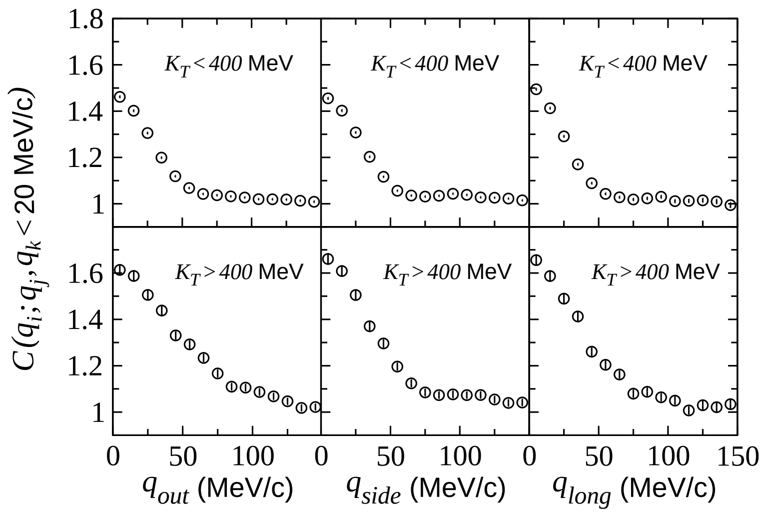

Fig. 3. Two-pion correlation functions for granular source.

increase the parameter $\Delta\mathcal{R}_t$, the variation of HBT radii $R_{\rm out}$ and $R_{\rm side}$ with K_T will become steep. As can be inferred from Fig. 4, the HBT results of granular source for $\Delta\mathcal{R}_t = 3.5$ fm agree quite well with experimental data, although the case with $\Delta\mathcal{R}_t = 0$ also give almost as good an agreement.

5. Conclusions and Discussion

The expansion of the dense matter (sQGP) produced in high-energy heavy-ion collisions at RHIC may be unstable. The initial transverse density may also be highly fluctuation. The unstable expansion and fluctuating initial transverse density may lead to a fragmentation of the system and the formation of a granular source of QGP droplets. Although a granular structure was suggested earlier as the signature of first-order phase transitions[26], the occurrence of granular structure may not be limited to the occurrence of first-order phase transitions. There are additional effects which may lead to the dynamical formation of granular droplets[18,21].

For a granular source of the droplets, the average particle emission time scales with the initial radius of the droplet. Pions will be emitted earlier if the radius is smaller. An earlier emission time will lead to a smaller extracted HBT radius $R_{\rm out}$, while the extracted HBT radius $R_{\rm side}$ is determined by the scale of the distribution of the droplets. A collective expansion of the droplets can further decrease $R_{\rm out}$ and the value of $R_{\rm out}/R_{\rm side}$ can be close to, or even less than 1 for an expanding granular source of QGP droplets.

In an event-mixing experimental analysis, some signals of the granular source such as the correlation function fluctuations may likely be suppressed after averaging over many events[19]. However, the property of an earlier average emission time

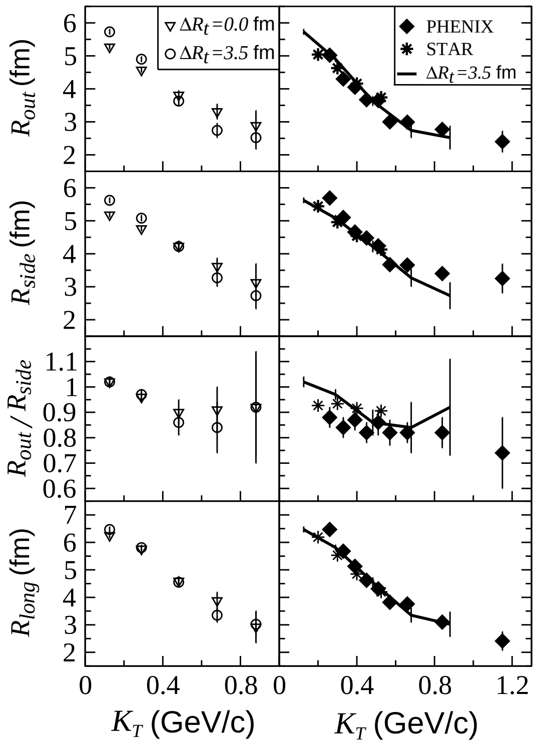

Fig. 4. Two-pion HBT radii obtained by the PHENIX Collaboration[6] and the STAR Collaboration[7] compared with the theoretical results calculated in the granular droplet model with $\Delta\mathcal{R}_t = 0$ and 3.5 fm.

of the granular source will remain as it is common to all events and not averaged out by event mixing. This property leads to a smaller $R_{\rm out}/R_{\rm side}$ in mixed-event HBT analysis.

In conclusion, a granular source model can explain the RHIC HBT puzzle and reproduce the data of pion HBT radii, as well as the data of pion transverse momentum spectra and elliptic flow[18] in $\sqrt{s_{NN}} = 200$ GeV Au + Au collisions at RHIC. It is of great interest to find direct evidences of the granular structure and to study the mechanics of granular source formation[17,18,19,20,21].

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